Title: Prime Time Secrets

Brief Overview:

In this unit, students will discover the meaning of factor, prime number, composite number, and prime factorization using the areas of rectangles that represent whole numbers. Students will learn these concepts by looking for patterns and making simple conjectures. The unit includes the Fundamental Theorem of Arithmetic using factor trees. Students use the Sieve of Eratosthenes to find the first 25 primes, and they explore the concept of Mersenne numbers. A simple crypto-system using prime factors is introduced to the students as a real world application of prime numbers.

NCTM 2000 Principles for School Mathematics:

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*
- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*
- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*
- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*
- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*
- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.*

Links to NCTM 2000 Standards:

- **Content Standards**

<u>Number and Operations</u>
Students will use the process of multiplication and division to find the prime factorization of whole numbers and to determine the first 25 prime numbers. They will use factors, multiples, and prime factorization to solve problems.

<u>Geometry</u>
Students will use their knowledge of area to determine factors of a number and to identify prime and students will construct rectangles and squares whose area represents two factors of specified numbers.

- **Process Standards**

Problem Solving
Students will develop a strategy for finding the factors of numbers using the area of rectangles. Students will develop a team strategy to determine if Mersenne Numbers are prime and to decrypt a secret message using a coding scheme based on primes.

Reasoning and Proof
Students will identify patterns and make conjectures about factors in order to discover the meaning of prime numbers, to determine the first 25 prime numbers, and to explore the Fundamental Theorem of Arithmetic.

Communication
Students will communicate both verbally and in writing the process used to find factors, primes, prime factors, and to break a secret code. Students will also compare and contrast the conjectures they made with those made by other students.

Connections
Students will associate their knowledge of area and multiplication with the process of identifying prime numbers. Students will also link their knowledge of prime numbers to crypto-systems used in banking and in other confidential communication.

Grade/Level:

6^{th} or 7^{th} grade General Math

Duration/Length:

This unit takes approximately five 45-minute class periods.

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Multiplication
- Division
- Area of rectangles and squares
- Exponents
- Divisibility rules up to twelve (optional)

Student Outcomes:

Students will:

- Identify all factors of a whole number.

- Determine if a number is prime, composite.
- Find the prime factorization of a composite number.
- Use a sieve method to find primes.
- Be able to describe what a prime number is and their significance.
- Communicate the process they used to solve problems.

Materials/Resources/Printed Materials:

- Square tiles (minimum of 30 per student pair)
- Colored pencils, crayons, or markers
- Blank transparencies and overhead markers (4 different colors)
- Scientific or four function calculators

Development/Procedures:

Day 1

- Introduce the unit and tell the students what they will be doing for the next week.
- Have the students complete Student Worksheet #1 in pairs.
- Put the students into groups of 4 to discuss the results on Student Worksheet #1.
- Have the students complete Follow-up Questions – Day 1 for homework.
- *Note- More specific instructions are given for each day later in the unit.

Day 2

- Put the students into different groups of 4 to discuss the observations they made during the homework assignment.
- Discuss as a class the results that were found on the previous night's homework.
- Introduce factor trees and work through the examples as a class.
- Have the students complete Student Worksheet #2 individually.
- Have the students complete Follow-up Questions – Day 2 at home.

Day 3

- Review the observation that the students made on the previous night's homework.
- Factor large numbers as a class and discuss observations.
- Give the students an Exit Question to answer before they leave class.

Day 4

- Discuss how many prime numbers there are and how one could find more prime numbers.
- Have the students complete Student Worksheet #3 in 4 groups.
- Discuss the results as a class.

- Introduce the concept of Mersenne Numbers and have the students find some Mersenne Numbers.
- Have the students complete the Follow-up Questions – Day 3 for homework.

Day 5

- Put the students into groups of 4 or 5.
- Tell the students some of the uses and history of crypto-systems.
- Pass out the Students Worksheet #4 and review the instructions.
- Begin the contest to find the secret code and award prizes to all groups who successfully decode the message, with a special prize going to the team that decodes it first.

Performance Assessment:

Students will be assessed daily on performance. Assessment will include teacher observation, in class activities, and the follow-up questions that the students will complete. Students will be asked to explain the process used to determine their answers during an in-class discussion and during the written activities. Exit questions may be also given at the end of the class periods as a formative assessment. The decoding activity at the end of the unit will provide an excellent assessment as well.

Extension/Follow Up:

- Take a field Trip to the National Cryptologic Museum.
- Use the prime factorization and Venn Diagrams to find the Greatest Common Factor and the Least Common Multiple.
- Initiate a class project to get involved with the Great Internet Mersenne Prime Search (GIMPS).
- Invite a speaker from the Mathematics Speakers Bureau to give a presentation on cryptology.

Authors:

Jamie Ankrom
Mill Creek Middle School
Calvert County, MD

Donald Link
Indian Creek School
Anne Arundel County, MD

Teacher Instructions

*Days are based on a 45 minute class period. Follow-up questions can be given in class if periods are longer than 45 minutes.

Day 1

- Introduce the topic to the class and briefly tell them what they will be accomplishing throughout the next week.
- Pass out both sheets of Student Worksheet #1 Representing Integers as Rectangles.
- Review the examples and discuss some strategies that the students might use to arrive at these answers. The students' goal is to create as many rectangles as possible using the stated number of square tiles. Tell the students that the rectangles such as 3×4 and 4×3 are considered the same for this activity. They will be recording the rectangles based on short side and long side, not length and width. Make sure that this is clear to them.
- Put the students into pairs and pass out the square tiles. Tell the students that they will be doing the same thing for all numbers from 2 to 30. They should find <u>all possible</u> rectangles that can be formed using the given number of tiles.
- When the students complete Student Worksheet #1 Representing Integers as Rectangles, allow them to consult with another pair. During this discussion students should compare answers and briefly talk about any patterns they see and conjectures about these patterns.
- For homework, give the students Follow-up Questions – Day 1. Each student should complete these questions individually to make conjectures and to identify patterns.

Day 2

- Put the students into groups of 4 (preferably not the same 4 as the previous day) and have them consult on the patterns that each student has found. Give the students ample time to discuss these findings.
- Begin a class discussion on the different patterns and conjectures that were found. Encourage students to explain how they arrived at their results. Students should have made the following observations. If students did not make these observations, then they should be prompted.
  - The product of the dimensions of the rectangle is equal to the number of square tiles used. More specifically, the number of square tiles used to make the rectangles is actually the area of the rectangle.
  - For each given number, the dimensions listed are all of the factors of the original number. Students may not use the word "factor" because they have not yet been introduced to it.
  - Some numbers only resulted in one rectangle, that is, the $1 \times n$ rectangle.
- Be sure to introduce the terms "factor," "prime number," and "composite number." Help the students define each.

- Tell the students that they will be using the factors that they found the previous day, but they will be breaking them down even further. Pass out Student Worksheet #2 – Page 1. Work through the examples with the students. Model the idea of using the rectangles from Student Worksheet #1 to aid the process. Emphasize that they cannot use the rectangles that have a dimension of 1. As you are working through the examples, tell the students that when they arrive at a number that cannot be broken down (without using the $1 \times n$ rectangle), they should either circle it or color it with a colored pencil.
- Pass out Student Worksheet #2 - Page 2 and Page 3. Have the students complete these pages individually. Students will be factoring some numbers in up to 3 different ways. Tell the students that if they are asked to do more than one factor tree for a given number, they should use different sets of factors to begin their tree with. Any work that the students do not finish in class should be given for homework.
- Follow-up Questions – Day 2 should be completed by the students at home. Again this is to help them notice patterns and make conjectures about their findings.

Day 3

- As a class review the observations that students made on Student Worksheet #2. Encourage each student to explain the process he or she used to arrive at their findings.
- Students should have observed the following items. If students did not make these observations, then they should be prompted.
  - Each colored or circled block is a prime number.
  - The product of the prime factors is the number itself.
  - The factor trees using different rectangular dimensions yield the same prime factors.
- Ask the students, "Can all whole numbers be represented as the product of prime numbers?" Discuss the students' ideas about this question.
- Factor some large numbers as a class (suggested examples are 156, 51, 1000, 53, 111) and as a class determine a strategy that could be used to attack such a problem. The included divisibility rules would be a good aid to these problems. While working through the problems, encourage the students to write the answers using exponents where possible (example, $36 = 2^2 \times 3^2$). A review of exponents might be needed.
- Discuss any observations and conclude by telling the students that the class has not <u>proven</u> that the conjecture is true for all numbers. Tell the students that the fact that the class has proven the conjecture true for all 7 or 8 numbers, does <u>not</u> mean that the conjecture is true for <u>all</u> numbers. A discussion about the definition of "conjecture" might be appropriate. To prove that it is true for all numbers would require math that they have not had yet, however it has been proven and the class can accept it as true for all numbers.
- Give the students the Exit Question. A suggested exit question is to have each student find the prime factorization of $72 = 2^3 \times 3^2$.

Day 4

- Begin the class by asking the students how many prime numbers they think there are, and how one could go about finding more of them. Discuss the thoughts that the students have regarding this question. Give the students Student Worksheet #3. As a class, read through the history of the Sieve Method. Split the class into 4 groups and give each group a transparency and a different colored marker. As a class they will work through the Sieve Method to determine the prime numbers from 2 to 100. Each group will be responsible for eliminating all of the numbers that have a certain prime number as a factor. For example, the first group will place the overhead over the sieve. This first group should color in all of the numbers that have 2 as a factor. The second group would do the same for 3, and other groups will follow for 5, and 7. Recommend to them that they use their hard paper copy as a rough draft and make their final draft on the transparency that you've given them. Remind the students that they should <u>not</u> color in the number 2, 3, 5, and 7 because we already know that they are prime.
- After each group finishes, have the first group come up and lay their coloring on the sieve. At this time that group should discuss how they accomplished their task and any patterns that they may have found. (A sieve has been included with the appropriate numbers colored for each prime). Repeat this for the other groups, having each group overlap their overheads. The end result should reveal the prime numbers from 2 to 100. There are 25 primes in this set.
- Again, ask the question "how many prime numbers they think there are." Lead this into a discussion of Mersenne (pronounced "mare-SEN") Numbers (2^n-1). Tell the students that this is a method used to find large prime numbers. Work through the examples of the Mersenne Numbers using 2, 3, and 4 as the exponent (n). Note that the exponents of 2 and 3 result in a prime number, but the exponent of 4 does not ($16 = 2 \times 8$).
- Tell the students that they will be doing the same thing for the exponents of 5, 6, 7, and 11. They are to find the Mersenne Numbers and to determine if these numbers are prime or composite. The first group to do this wins a prize. Students should be allowed to use calculators and work in groups. Note that the Mersenne Numbers for 6 and 11 are <u>not</u> prime. (An exponent of 6 results in 63 which can be factored into 7×9, and an exponent of 11 results in 2047 which can be factored into 23×89.)
- Have the students complete Follow-up Questions – Day 3 for homework. Be sure to emphasize that they are to write a <u>phone</u> conversation between themselves and a friend explaining the concept of prime numbers and prime factorization.

Day 5

- Put the class into teams of 4 or 5 for The Great Secret Code Challenge (see Student Worksheet #4). Discuss the need for secret codes and tell students that some of the most successful crypto-systems use prime numbers to make breaking codes very difficult.
- Hand out Student Worksheet #4 and go over the instructions and make sure every team understands what they need to do. It might be beneficial to show an example of factoring a number and summing the prime factors. Note that students may use calculators. Encourage students to use their divisibility rules as an aid also.

- When everyone is ready, begin the contest, indicating that every team that gets the correct answer will receive a prize and that the first team to finish with the correct answer will get a special prize. The coded message is PRIMESECRETS.
  - The Secret Key of 1505 has prime factors 5 x 7 x 43, so the Secret Factor is 43.
  - Here's a few examples of decoding code numbers according to the student instructions:

Code Number:	082	048	133
Subtract the Secret Factor:	- 43	- 43	- 43
Result after Subtracting:	39	5	90
Find Prime Factors:	3 × 13	5	2 × 3 × 3 × 5
Sum the Prime Factors:	16	5	13
Convert to a Letter:	P	E	M

- Give out the prizes and have a class discussion about how each team approached the challenge.
- Explain to the class that in real crypto-systems used today, the keys used are very large numbers (e.g., 100 digits). To break the code, it is necessary to find the prime factors of these large numbers, which is next to impossible when the number is so large.

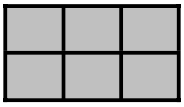
Whole Number
Divisibility Rules

Divisible by:	If:
2	Last digit is even or 0
3	Sum of digits is a multiple of 3
4	Last two digits are a multiple of 4 or 0
5	Last digit is 0 or 5
6	Divisible by 2 and 3
7	Double the last digit and subtract it from the number represented by the remaining digits. If the answer is 0 or divisible by 7, then the original is divisible by 7.
8	Divisible by 2 and 4
9	Sum of digits is a multiple of 9
10	Last digit is 0
11	Subtract the sum of the even digits from the sum of the odd digits; if the difference, including 0, is divisible by 11, the number is also.
12	Divisible by 3 and 4

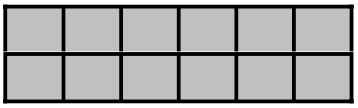
Name _____

Student Worksheet #1
Representing Integers as Rectangles
Examples

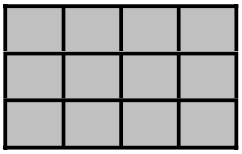
6 can be represented as a 2 by 3 rectangle:



12 can be represented as a 2 by 6 rectangle:



or as a 3 by 4 rectangle:



Filling Out the Student Worksheet

	Rectangle #1		**Rectangle #2** (If there is one)		**Rectangle #3** (If there is one)		**Rectangle #4** (If there is one)	
# of tiles	**Short Side**	**Long Side**	**Short Side**	**Long Side**	**Short Side**	**Long Side**	**Short Side**	**Long Side**
6	1	6	2	3				
12	1	12	2	6	3	4		

Name _____

Student Worksheet #1
Representing Integers as Rectangles

	Rectangle #1		**Rectangle #2** (If there is one)		**Rectangle #3** (If there is one)		**Rectangle #4** (If there is one)	
# of tiles	**Short Side**	**Long Side**	**Short Side**	**Long Side**	**Short Side**	**Long Side**	**Short Side**	**Long Side**
2								
3								
4								
5								
6	1	6	2	3				
7								
8								
9								
10								
11								
12	1	12	2	6	3	4		
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								

Worksheet #1
Representing Integers as Rectangles
Student Worksheet (ANSWERS)

	Rectangle #1		**Rectangle #2** (If there is one)		**Rectangle #3** (If there is one)		**Rectangle #4** (If there is one)	
# of tiles	**Short Side**	**Long Side**	**Short Side**	**Long Side**	**Short Side**	**Long Side**	**Short Side**	**Long Side**
2	**1**	**2**						
3	**1**	**3**						
4	**1**	**4**	**2**	**2**				
5	**1**	**5**						
6	**1**	**6**	**2**	**3**				
7	**1**	**7**						
8	**1**	**8**	**2**	**4**				
9	**1**	**9**	**3**	**3**				
10	**1**	**10**	**2**	**5**				
11	**1**	**11**						
12	**1**	**12**	**2**	**6**	**3**	**4**		
13	**1**	**13**						
14	**1**	**14**	**2**	**7**				
15	**1**	**15**	**3**	**5**				
16	**1**	**16**	**2**	**8**	**4**	**4**		
17	**1**	**17**						
18	**1**	**18**	**2**	**9**	**3**	**6**		
19	**1**	**19**						
20	**1**	**20**	**2**	**10**	**4**	**5**		
21	**1**	**21**	**3**	**7**				
22	**1**	**22**	**2**	**11**				
23	**1**	**23**						
24	**1**	**24**	**2**	**12**	**3**	**8**	**4**	**6**
25	**1**	**25**	**5**	**5**				
26	**1**	**26**	**2**	**13**				
27	**1**	**27**	**3**	**9**				
28	**1**	**28**	**2**	**14**	**4**	**7**		
29	**1**	**29**						
30	**1**	**30**	**2**	**15**	**3**	**10**	**5**	**6**

Name _____

Follow Up Questions- Day 1

1. What patterns do you see among the number of rectangles formed and the number of tiles that were used on Worksheet #1? Use complete sentences.

_____

_____

_____

_____

_____

_____

2. How do the dimensions (length and width) of each rectangle relate to the number of tiles used? Use complete sentences.

_____

_____

_____

3. Fill in the rectangle dimensions for the following numbers:

	Rectangle #1		**Rectangle #2** (If there is one)		**Rectangle #3** (If there is one)		**Rectangle #4** (If there is one)	
# of tiles	**Short Side**	**Long Side**	**Short Side**	**Long Side**	**Short Side**	**Long Side**	**Short Side**	**Long Side**
43	1	43						
100	1	100						
77	1	77						

Scoring Rubric for Follow-up Questions – Day 1

Number 1:

3- A student will receive a score of 3 if the student
- correctly described at least two patterns (e.g., some numbers have only one rectangular representation, some have two or three different representations, perfect squares have a square representation);
- used good sentence structure with three or fewer grammatical errors.

2- A student will receive a score of 2 if the student
- correctly described at least one pattern;
- used fair sentence structure with five or fewer grammatical errors.

1- A student will receive a score of 1 if the student
- did not identify any patterns correctly;
- used poor sentence structure with more than five grammatical errors.

0- A student will receive a score of 0 if the student
- did not complete the problem or did not turn in the activity.

Scoring Rubric for Follow-up Questions – Day 1

Number 2:

A student will receive a score of 3 if the student
correctly indicated that the dimensions of all the

- grammatical errors.

2- A student will receive a score of 2 if the student
- correctly described some relationship between the numbers;
  used fair sentence structure with five or fewer grammatical errors.

1- A student will receive a score of 1 if the student
- did not describe any correct relationship;
- used poor sentence structure with more than five grammatical errors.

0- A student will receive a score of 0 if the student
- did not complete the problem or did not turn in the activity.

Scoring Rubric for Follow-up Questions – Day 1

Number 3:

3- A student will receive a score of 3 if the student
- correctly determined the factor rectangles for all three numbers (43: 1x43, 100: 2x50, 4x25, 5x20, 77: 7x11).

2- A student will receive a score of 2 if the student
- correctly determined the factor rectangles for two of the numbers.

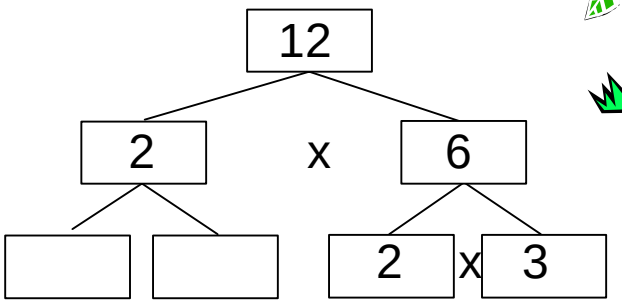
1- A student will receive a score of 1 if the student
- correctly determined the factor rectangles for one of the numbers.

0- A student will receive a score of 0 if the student
- did not complete the problem or did not turn in the activity.

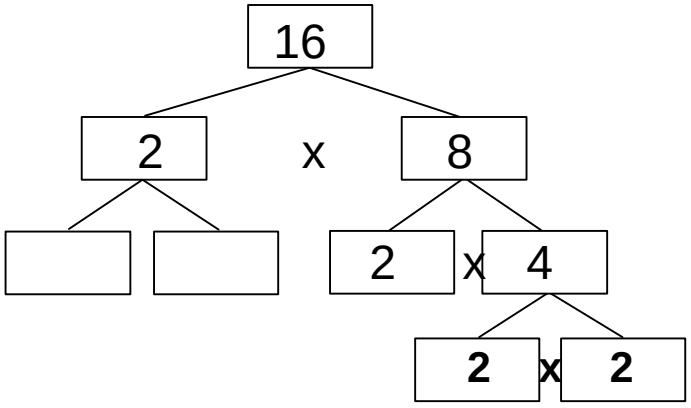
Name _____

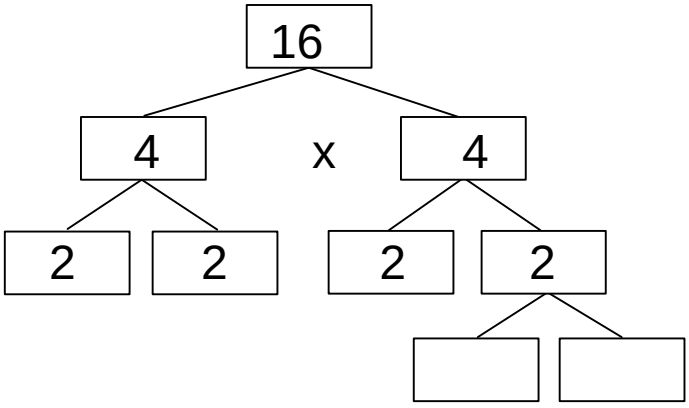
Student Worksheet #2 - Page 1
Factor Trees

Examples



2 2 3 ____ ____



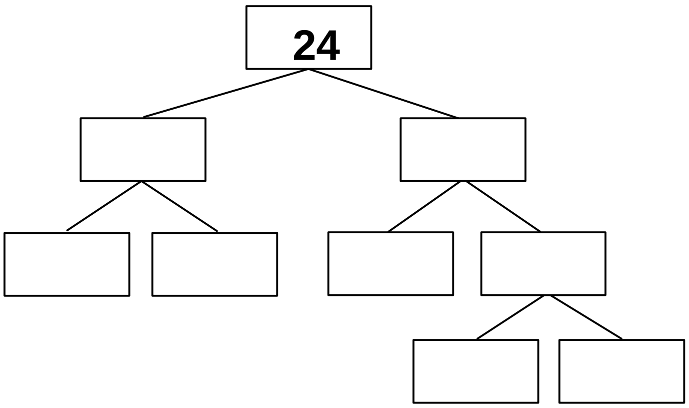


2 2 2 2 ____ 2 2 2 2 ____

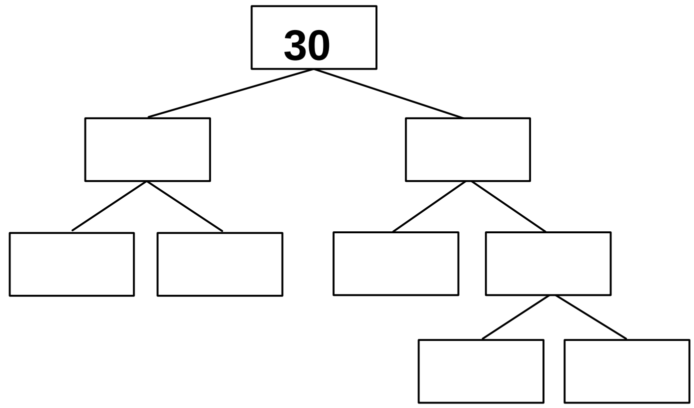
Name ____________________

Student Worksheet #2 - Page 2
Factor Trees

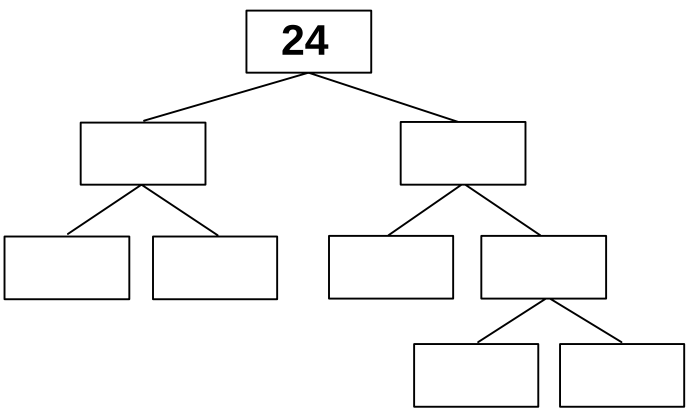
Factor each of these numbers, using the factors from your Worksheet #1. If you are asked to create more than one factor tree for a number, choose different factors to begin your tree with. Color or circle the last number on each branch of each tree. Then copy those numbers into the blank spaces below each tree.



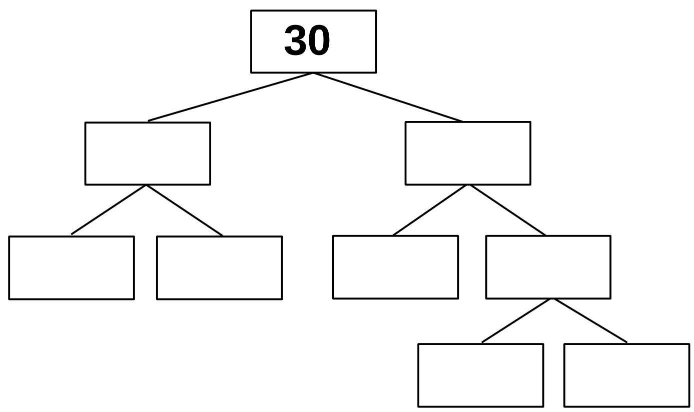
_____ _____ _____ _____ _____



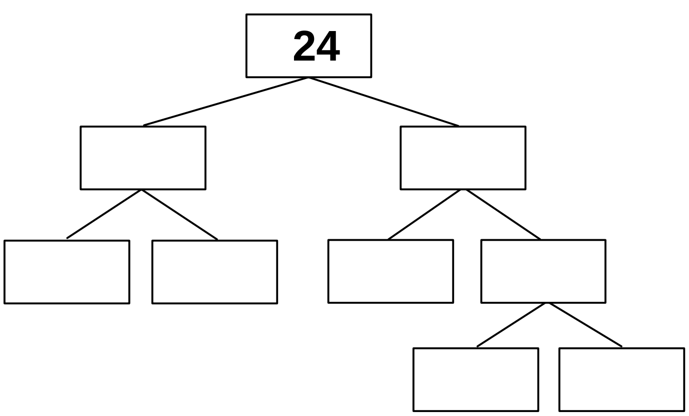
_____ _____ _____ _____ _____



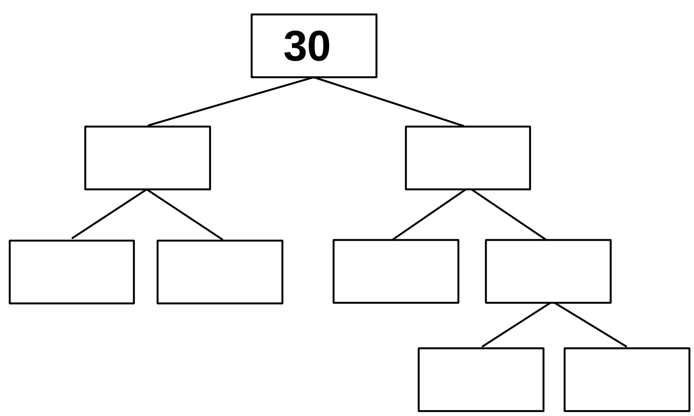
_____ _____ _____ _____ _____



_____ _____ _____ _____ _____



_____ _____ _____ _____ _____



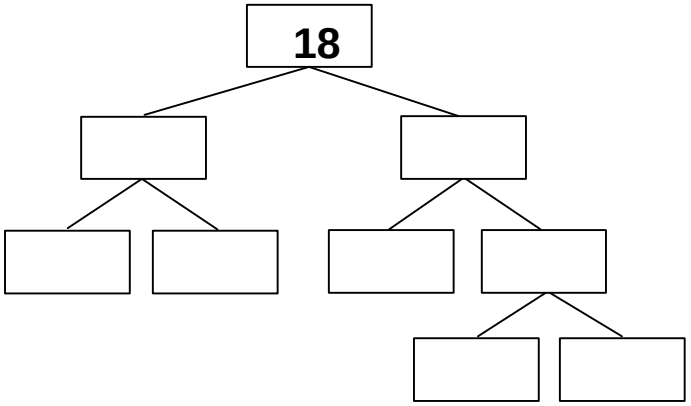
_____ _____ _____ _____ _____

Name _____

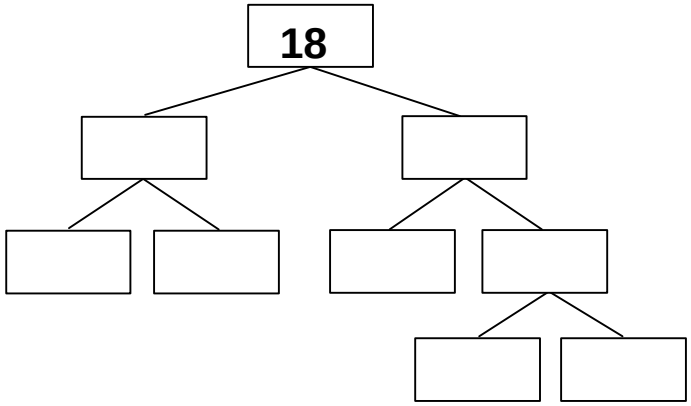
Student Worksheet #2 – Page 3

Factor Trees

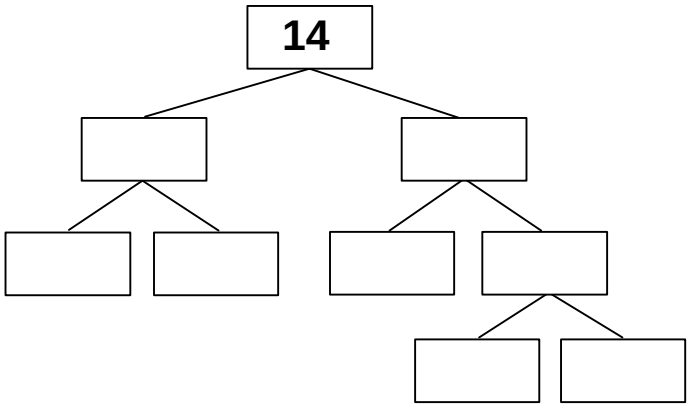
Factor each of these numbers, using the factors from your Worksheet #1 to complete the factor trees. . If you are asked to create more than one factor tree for a number, choose different factors to begin your tree with.



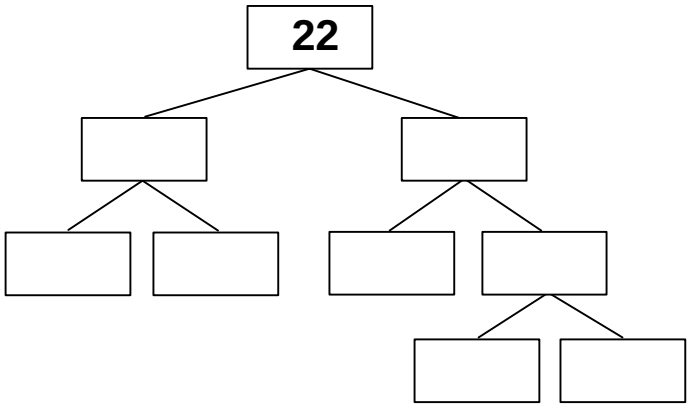
____ ____ ____ ____ ____



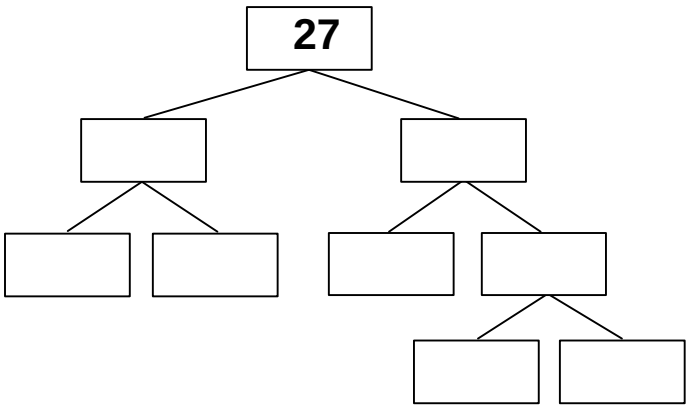
____ ____ ____ ____ ____



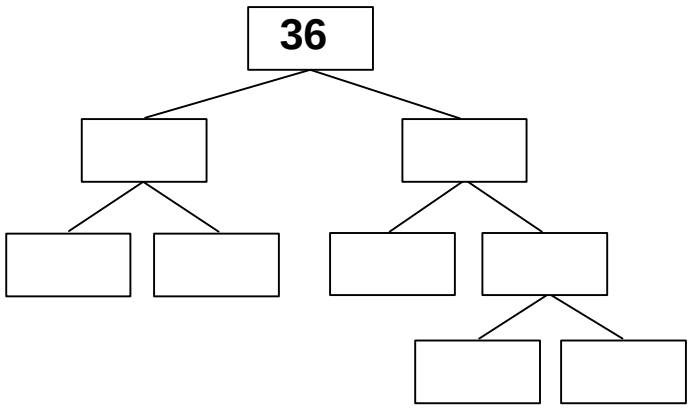
____ ____ ____ ____ ____



____ ____ ____ ____ ____



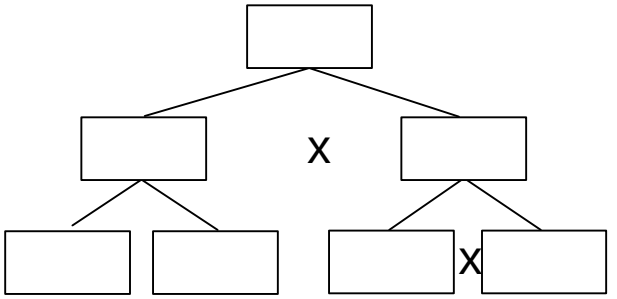
____ ____ ____ ____ ____



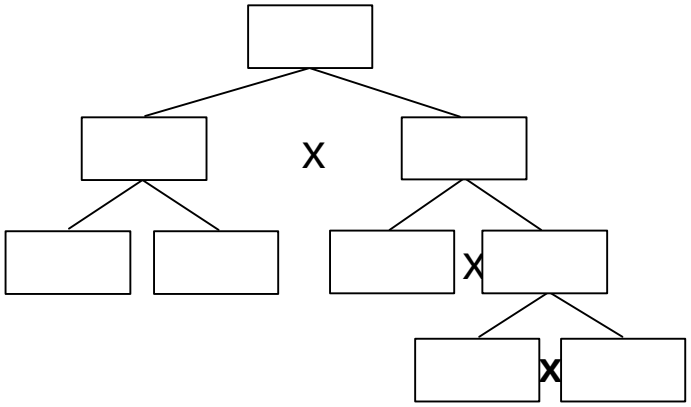
____ ____ ____ ____ ____

Student Worksheet #2 – Page 1
Factor Trees (ANSWERS)

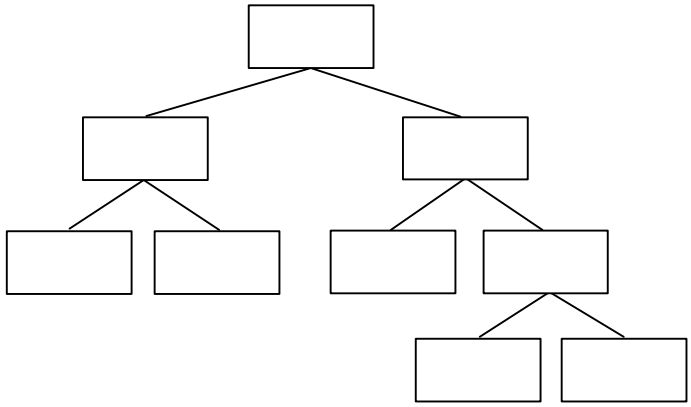
Examples



2 2 3 ____ ____



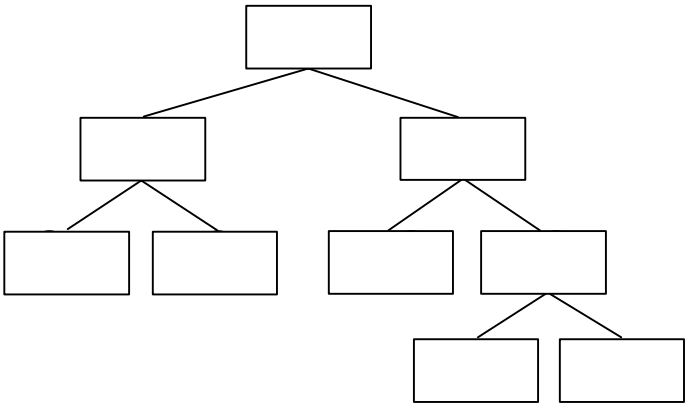
2 2 2 2 ____



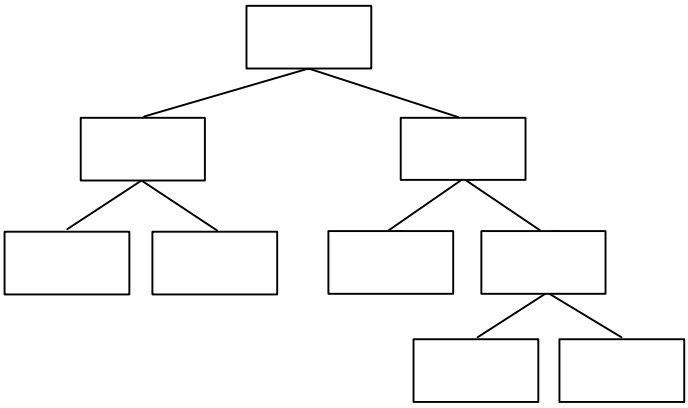
2 2 2 2 ____

Student Worksheet #2 – Page 2
Factor Trees (ANSWERS)

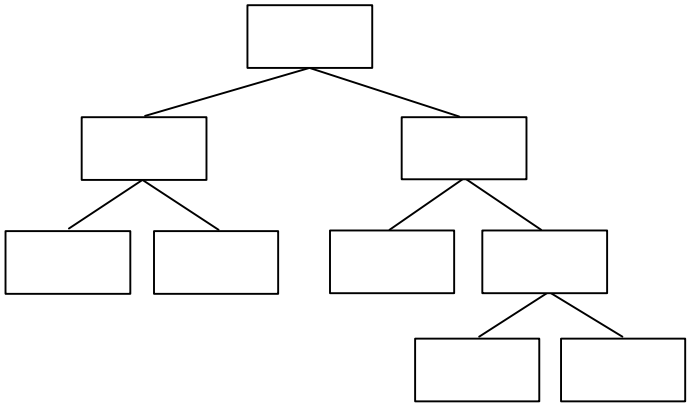
Factor each of these numbers, using the factors from your Worksheet #1. If you are asked to create more than one factor tree for a number, choose different factors to begin your tree with. Color or circle the last number on each branch of each tree. Then copy those numbers into the blank spaces below each tree.



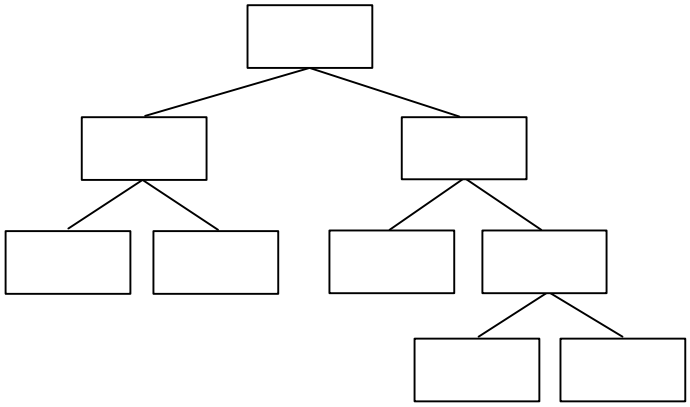
2 2 2 3 ____



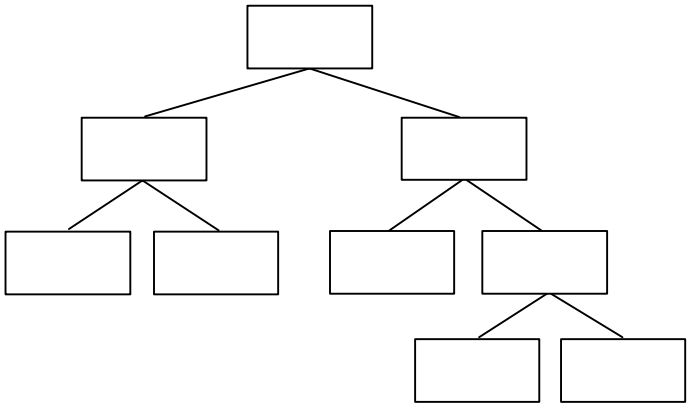
5 2 3 ____ ____



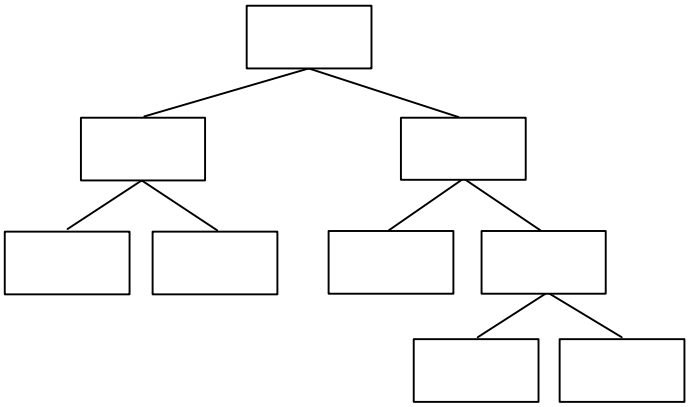
3 2 2 2 ____



2 3 5 ____ ____



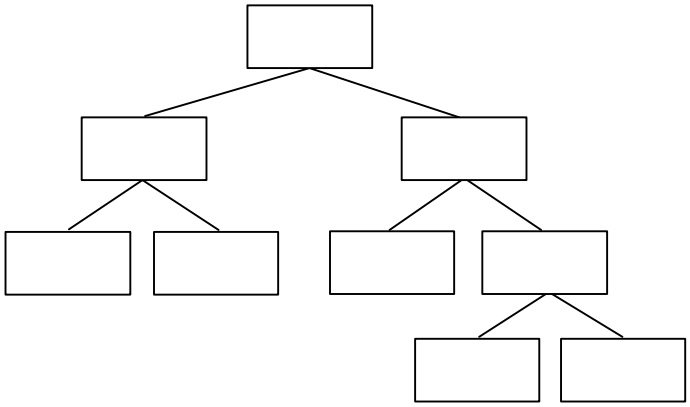
2 3 2 2 ____



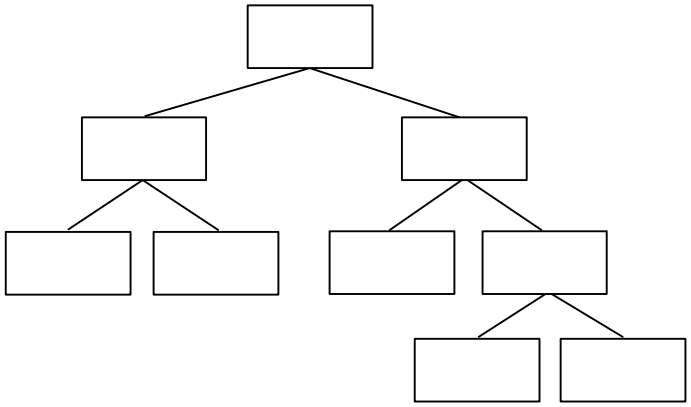
3 2 5 ____ ____

Student Worksheet #2 – Page 3
Factor Trees (ANSWERS)

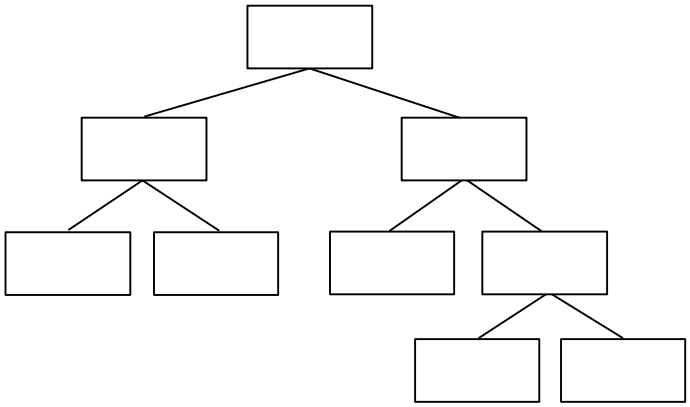
Factor each of these numbers, using the factors from your Worksheet #1 to complete the factor trees. If you are asked to create more than one factor tree for a number, choose different factors to begin your tree with.



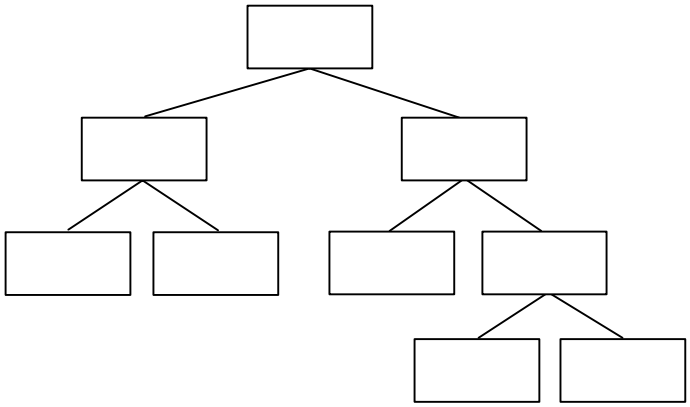
3 2 3 ____ ____



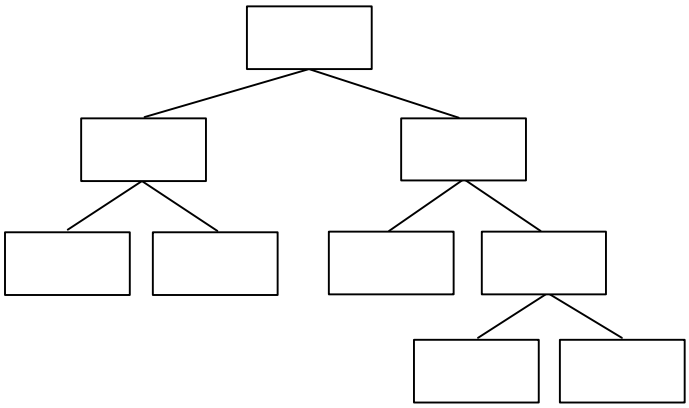
2 3 3 ____ ____



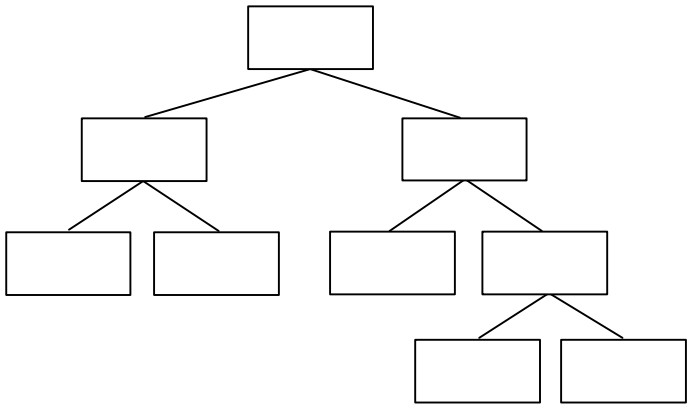
2 7 ____ ____ ____



2 11 ____ ____ ____



3 3 3 ____ ____



2 2 3 3 ____

Name _____

Follow Up Questions- Day 2

1. How do the numbers on the ends of each tree's branches relate to the number at the top of the tree? Use complete sentences.

2. How do the numbers on the ends of each tree's branches relate to the patterns we saw with the number of rectangles formed on Worksheet #1? Use complete sentences.

3. How do the numbers on the ends of the branches for the three factor trees you did for the number 30 compare with each other? Use complete sentences.

Scoring Rubric for Follow-up Questions – Day 2

Number 1:

3- A student will receive a score of 3 if the student
- correctly indicated that the product of the leaves equals the tree number;
- used good sentence structure with three or fewer grammatical errors.

2- A student will receive a score of 2 if the student
- correctly indicates that the product of the leaves equals the tree number;
- used fair sentence structure with four or five grammatical errors.

1- A student will receive a score of 1 if the student
- did not correctly state the connection;
- used poor sentence structure with more than five grammatical errors.

0- A student will receive a score of 0 if the student
- did not complete the problem or did not turn in the activity.

Scoring Rubric for Follow-up Questions- Day 2

Number 2:

3- A student will receive a score of 3 if the student
- correctly indicated that the leaves are all prime numbers;
- used good sentence structure with three or fewer grammatical errors.

2- A student will receive a score of 2 if the student
- correctly indicated that the leaves are all prime numbers;
- used fair sentence structure with four or five grammatical errors.

1- A student will receive a score of 1 if the student
- did not describe any correct connection;
- used poor sentence structure with more than five grammatical errors.

0- A student will receive a score of 0 if the student
- did not complete the problem or did not turn in the activity.

Scoring Rubric for Follow-up Questions – Day 2

Number 3:

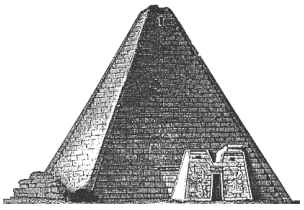
3- A student will receive a score of 3 if the student
- correctly indicated that the leaves all contained the same set of numbers;
- used good sentence structure with three or fewer grammatical errors.

2- A student will receive a score of 2 if the student
- correctly indicated that the leaves all contained the same set of numbers;
- used fair sentence structure with four or five grammatical errors.

1- A student will receive a score of 1 if the student
- did not describe any correct connection;
- used poor sentence structure with more than five grammatical errors.

0- A student will receive a score of 0 if the student
- did not complete the problem or did not turn in the activity.

Name _____



Student Worksheet #3
The Sieve of Eratosthenes

Eratosthenes was born around 276 C.E. in Cyrene, which is now in Libya in North Africa. He was the first person to accurately estimate the diameter of the earth (accurate to within 1%). He made many other major contributions to the progress of science, including a calendar that used leap years and a star chart containing 675 stars.

Eratosthenes also invented a method for efficiently constructing tables of prime numbers, called the "Sieve of Eratosthenes."

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Student Worksheet #3
Answers: Multiples of 2

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Student Worksheet #3
Answers: Multiples of 3

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Student Worksheet #3
Answers: Multiples of 5

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Student Worksheet #3
Answers: Multiples of 7

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Name _____

Student Handout
Sieve of Eratosthenes
Primes Between 1 and 100

	2	3		5		7			
11		13				17		19	
		23						29	
31						37			
41		43				47			
		53						59	
61						67			
71		73						79	
		83						89	
						97			

Name _____

Follow Up Questions – Day 3

Imagine that a good friend calls you up on the phone. This friend tells you that he or she has been absent for the last 4 days and is very concerned about missing Math class. Your friend asks you to explain (in detail) what you have discussed the past few days. Write the phone conversation that would occur as you explain to your friend factors, prime and composite numbers, and the prime factorization of a number. Be sure to use correct terminology. Remember this is your friend. You want to make sure that he or she has an excellent understanding of the topics. (Your friend may have any name you choose).

Scoring Rubric for Follow-up Questions – Day 3

3- A student will receive a score of 3 if the student
- used appropriate terminology;
- gave clear and correct verbal explanations of the concept;
- exhibited good sentence structure and made three or fewer grammatical errors.

2- A student will receive a score of 2 if the student
- used most terminology appropriately;
- gave a clear and correct verbal explanation of at least one concept;
- used good sentence structure and made fewer than five grammatical errors.

1- A student will receive a score of 1 if the student
- used terminology incorrectly;
- gave a poor or incorrect explanation of the concept;
- used poor sentence structure and made more than five grammatical errors.

0- A student will receive a score of 0 if the student
- did not complete the activity or did not turn in the activity.

Student Worksheet #4
The Great Secret Code Challenge

Banks, businesses, governments, and spies must be able to send secret messages around the world. They need a way to scramble (encrypt) each message in such a way that only the intended recipient can unscramble (decrypt) it.

Some of the most successful encrypting systems use prime numbers and factoring to keep the messages secret. Here s an example:

Code Numbers:	**082**	**108**	**057**	**133**	**049**	**077**	**048**	**046**	**120**	**049**	**094**	**160**
Actual Message:	___	___	___	___	___	___	___	___	___	___	___	___

Your team is to decrypt the above message. Fortunately, one of your operatives has intercepted the secret <u>key</u> that is needed to unlock the message. It is 1505. To decode the message, follow these steps:

1. Factor the secret key into its prime factors and record the largest one here:

   Largest Prime Factor: _____ (we'll call this the *Secret Factor*)

2. For each 3-digit code number in the coded message above, do this:

   a) Subtract the Secret Factor from the code number.

   b) Factor the resulting number into its prime factors (<u>remember not to use 1 as a prime factor</u>).

   c) Add the prime factors together.

   d) The resulting number represents a single letter in the original message (1=A, 2=B, 3=C, … , 26=Z)

3. Write the corresponding letter in the blank under each code number to unlock the original message.

YOU MAY USE A CALCULATOR